RAPID COMMUNICATIONS

PHYSICAL REVIEW E **73**, 015601(R) (2006)

Incoherent solitons in instantaneous nonlocal nonlinear media

Oren Cohen,[1] Hrvoje Buljan,[1,2] Tal Schwartz,[1] Jason W. Fleischer,[1,3] and Mordechai Segev[1]

[1]*Physics Department, Technion—Israel Institute of Technology, Haifa 32000, Israel*

[2]*Department of Physics, University of Zagreb, PP 322, 10000, Zagreb, Croatia*

[3]*Electrical Engineering Department, Princeton University, New Jersey 08544*

(Received 9 September 2004; published 26 January 2006)

We predict random-phase spatial solitons in instantaneous nonlocal nonlinear media. The key mechanism responsible for self-trapping of such incoherent wave packets is played by the nonlocal (rather than the traditional noninstantaneous) nature of the nonlinearity. This kind of incoherent soliton has profoundly different features than other incoherent solitons.

DOI: 10.1103/PhysRevE.73.015601 PACS number(s): 42.65.Tg, 42.65.Jx

The observation of spatially incoherent optical solitons [1] has opened up a new direction in nonlinear science [1–8]. Such spatially incoherent solitons—self-trapped entities whose structure varies randomly in time—form when diffraction, governed by the beam's correlation function, is robustly balanced by nonlinear self-focusing. This balance results in the stationary propagation of the beam's envelope (i.e., its time-averaged intensity) [3]. A prerequisite for the formation of spatially incoherent solitons is that the nonlinearity responds to the envelope of the beam rather than to the fluctuating intensity pattern. Otherwise, the speckled nature of the field would induce intricate spatial variations in the refractive index, causing beam fragmentation and prohibiting self-trapping. In their original concept, incoherent solitons were studied with a noninstantaneous nonlinearity having a response time, τ, much longer than the characteristic beam fluctuation time $t_c (\tau \gg t_c)$. There, the nonlinearity time averages over the stochastic multimode character of the instantaneous speckled field [3], responding only to the envelope of the beam. In fact, spatially incoherent solitons have been studied only in rather slow nonlinear media [1–5], and it has been believed that a noninstantaneous response is a mandatory condition for the formation of such solitons [3,6]. Here, we show that if the nonlinearity has a *nonlocal* nature, it can filter out the otherwise highly fragmented variations in the refractive index induced by the rapidly fluctuating multimode field. In this fashion, *incoherent (random-phase) spatial solitons can form in instantaneous nonlocal nonlinear media*.

Nonlocal nonlinearities are inherent to many systems, when the underlying mechanism involves transport (of heat [9], atoms in a gas [10], charge carriers in semiconductors [11], etc.), long-range forces (e.g., electrostatic interactions in liquid crystals [12]), or photon attraction [13]. Nonlocality also affects the propagation of waves in plasma [14], and matter waves in Bose-Einstein condensates, where nonlocality arises from the underlying many-body interactions [15]. For localized wave packets, of which solitons are an exemplary phenomenon, nonlocality becomes important when the range of nonlocal interactions is appreciable on the scale of the variations in the beam profile. Nonlocality has profound consequences on solitons [16–20] by arresting catastrophic collapse [17], suppressing azimuthal instability [18,19], and giving rise to attractions between dark solitons (that otherwise repel) [20], etc.

Here, we predict a new type of incoherent soliton, forming in a fast nonlocal nonlinear medium, i.e., a medium responding much faster than the characteristic fluctuation time $(t_c \gg \tau)$. Such "instantaneous" incoherent solitons form when (i) the beam is self-trapped within a time frame much shorter than t_c, and (ii) the transverse momentum of the beam is constant in time. When the latter condition is violated, the time-averaged intensity of the beam exhibits a new type of a propagation-broadening mechanism: *Statistical nonlinear diffraction*.

The propagation of weakly correlated waves in the fast-responding nonlocal nonlinear media is an issue of generic interest. Here, we analyze this problem in the context of optics. Consider a quasimonochromatic partially spatially incoherent beam, propagating in a fast-responding nonlocal nonlinear medium. The characteristic speckle size ($\approx$transverse correlation distance) is at least several times larger than the wavelength; hence the paraxial approximation is valid. The characteristic time scales involved are τ, t_c, and the "time of flight" τ_f during which a light beam passes through the medium. Here, we consider the regime in which $\tau \ll t_c$ and $\tau_f \ll t_c$. For the sources typically used to study random-phase solitons (e.g., rotating diffusers [1]), the coherence time is a controllable variable. Hence, the relations above and the nonlinear dynamics we suggest below are experimentally accessible in all nonlocal nonlinearities. This said, especially attractive are very fast highly nonlocal nonlinearities that were so far believed to be inaccessible for supporting random-phase solitons, simply because their response would not average out the random intensity fluctuations. For example, with the ideas presented here, random-phase solitons can be generated in semiconductors [11], in which the self-focusing on/off response time is associated with the recombination time of the charge carriers (typically subnanosec), and is also highly nonlocal due to the charge carriers high mobility [11].

The complex field $\Psi(x,z,t)$ describing the spatially incoherent light is fluctuating with a characteristic time scale t_c. We study the propagation of $\Psi(x,z,t)$ in two steps. First, we analyze the propagation within a very short time interval $(\ll t_c)$ during which the beam can be treated as a coherent speckled (multimode) wave. Second, we calculate the propagation of the time-averaged envelope [21]. Assuming that the incoherent light source is ergodic, the time average corresponds to an ensemble average over possible realizations of the speckled field. In addition, because the response time of the nonlinear medium is very short, the nonlinearity has no memory, and we resort to ensemble averaging while analyzing the propagation of the (time-averaged) beam envelope.

First we study the dynamics within the short time frame. The dimensionless, slowly varying amplitude of the optical field $\Psi(x,z,t)$ evolves according to [22]

$$i\frac{\partial\Psi}{\partial z}+\frac{\partial^2\Psi}{\partial x^2}+\Delta n[|\Psi|^2]\Psi(x,z,t)=0, \quad (1)$$

where x and z are the dimensionless transverse and propagation directions, respectively. The nonlocal nonlinear term Δn has the form of a spatial convolution between the instantaneous wave intensity $|\Psi(x,z,t)|^2$ and the response function $R(x,x')=R(x-x')$ of the medium [17]

$$\Delta n(x,z,t)=\int_{-\infty}^{\infty}R(x'-x)|\Psi(x',z,t)|^2dx'. \quad (2)$$

For concreteness, consider a Gaussian response function $R=1/\sqrt{\pi\sigma^2}\exp[-(x-x')^2/\sigma^2]$. Here, we are interested in the highly nonlocal regime, i.e., the width of the response function σ is much larger than the width of the beam. In this regime, the nonlinear index change averages over the variations in the beam profile, and Δn has a approximately parabolic shape, depending only on the total power [16,17]

$$\Delta n(x\,.\,z\,.\,t)\approx\frac{P(t)}{\sqrt{\pi\sigma^2}}\left\{1-\frac{[x-a(z,t)]^2}{\sigma^2}\right\}, \quad (3)$$

where $P(t)=\int_{-\infty}^{\infty}|\Psi(x,z,t)|^2dx$ is the total power of the beam within this time frame, and $a(z,t)=\int x|\Psi(x,z,t)|^2dx$ is the "beam center" at plane z. The center of the induced waveguide coincides with the center of the beam $a(z,t)$. The beam enters the nonlinear medium at angle $\theta(t)$ (with respect to z) which varies stochastically with time. This propagation angle corresponds to the initial transverse momentum of the beam: $\theta(t)=2\bar{k}_x(t)=2\int_{-\infty}^{\infty}k_x|\tilde{\Psi}(k_x,z=0,t)|^2dk_x$ (the factor 2 appears because in our dimensionless units, $k_z=\frac{1}{2}$). Because Eq. (1) conserves transverse momentum, the angle does not change along z, and the center of the waveguide $a(z,t)$ lies on a straight line: $a(z,t)=a(0,t)+2\bar{k}_x(t)z$.

The discussion above assumes that the width of the beam is much smaller than the nonlocality range σ for all z. Let us examine when this happens. Consider a beam $\Psi(x,z=0,t)$ that at $z=0$ is much narrower than the nonlocality range σ. In this limit, the beam induces a parabolic waveguide at the vicinity of the input face, and some of its guided modes are excited by $\Psi(x,z=0,t)$. If all the modes excited by the beam are much narrower than σ, the highly nonlocal limit is satisfied throughout the propagation. In this situation, the instantaneous induced waveguide is stationary with a straight line trajectory of $a(z,t)$, while the beam is populating its guided modes in a self-consistent fashion [23]. The instantaneous beam is thus self-trapped, yet its intensity oscillates periodically due to "beating" among the modes comprising it.

Interestingly, having a self-trapped speckled beam at any instantaneous time frame does not necessarily guarantee that the time-averaged behavior of such a beam exhibits self-trapping. This is because the initial propagation angle of the beam, $\theta(t)$, and its transverse displacement $a(0,t)$, fluctuate randomly on time-scale t_c. We, therefore, examine the time-average behavior of the system, with the average taken over $t\gg t_c$. From ergodicity, such averaging is equal to an ensemble average over all possible initial conditions $\theta(t)$ and $a(0,t)$. Let us denote $p(\bar{k}_x)$ as the probability distribution of the transverse momentum of the incident beam. Consider first the case where $\bar{k}_x(t)$ is a random variable, hence $p(\bar{k}_x)$ has some width. When self-consistency is satisfied (the beam is self-trapped in each frame), the instantaneous intensity at a large enough z is located in the vicinity of the beam center, $a(z,t)\approx 2\bar{k}_x(t)z$. Thus, the time-averaged intensity $\langle|\psi(x,z,t)|^2\rangle$ after a large distance z assumes the shape of $p(\bar{k}_x)z$. That is, the time-averaged intensity *broadens*, with a width proportional to z and to the width of the probability distribution of the transverse momentum of the light (defined by the source) $p(\bar{k}_x)$. Consequently, an incoherent beam in a nonlocal nonlinear medium may form self-trapped solitonic beams in each short time frame, while its time-averaged intensity structure is broadening. We emphasize that this propagation broadening is nonlinear, arising because an incoherent source typically emits light with stochastically varying directionality. Henceforth we address this new propagation-broadening mechanism as a *statistical nonlinear diffraction*.

There are cases, however, when the statistical nonlinear diffraction is eliminated. Such cases occur, for example, when the source emitting the incoherent light does not have randomly fluctuating transverse momentum, i.e., when $p(\bar{k}_x)=\delta(\bar{k}_x)$, thus forming an instantaneous self-trapped beam with $\theta=0$ at all times t. In this case, the beam self-traps within each short time frame and also forms a time-averaged random-phase soliton.

Let us now analyze some examples. Consider first a beam which at $z=0$ is a superposition of two uncorrelated coherent Gauss-Hermite waves [24]

$$\Psi(x,t,z=0)=\frac{\exp(-x^2)}{\sqrt[4]{2\pi}}\left[\sqrt{\frac{P_n}{2^n n!}}H_n(\sqrt{2}x)+\sqrt{\frac{P_m}{2^m m!}}H_m(\sqrt{2}x)\exp(i\varphi(t))\right], \quad (4)$$

where $n\neq m$, H_n is the Hermite polynomial of order n, P_n is the modal power of wave n, and $\varphi(t)$ is a real random variable uniformly distributed in the interval $\{-\pi,\pi\}$. Such a beam, with P_n and P_m being constants and $\varphi(t)$ stochastic, has been used in the past to generate multimode solitons [25]. In the highly nonlocal limit, this beam induces a parabolic waveguide whose width depends only on the total power $P=P_n+P_m$. First, we set $P_n=P_m=2\sqrt{\pi}\sigma^3$. For such P values, the uncorrelated waves of $\Psi(x,z=0,t)$ coincide with the guided (Gauss-Hermite) modes of the induced waveguide. To work out the time-averaged propagation of such a beam in our system, we average over the propagation of 100 realizations ($\phi_j=-\pi+2\pi\, j/100\;\; j=1,2,3\ldots,100$). In each frame, we simulate the evolution of the coherent beam, Ψ_i, [via Eq. (1)], assuming a Gaussian response function [via Eq. (2)] with $\sigma=20$.

Figure 1 presents the results with $n=0$ and $m=1$. Figures 1(a) and 1(b) show two representative frames: $\varphi=0$ [Fig. 1(a)] and $\varphi=\pi/2$ [Fig. 1(b)]. Within each frame, the beam is self-trapped, yet the instantaneous beams are propagating

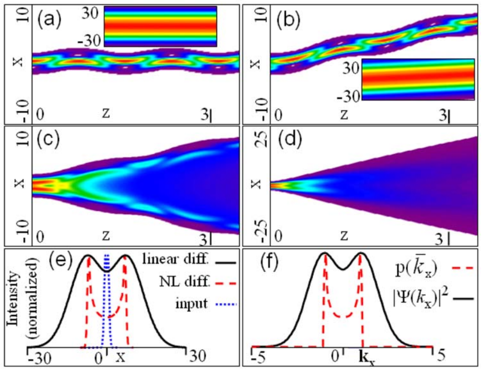

FIG. 1. (Color online) Propagation of a random-phase beam comprising of Gauss-Hermite waves 0 and 1. Single frame self-trapped propagation when (a) $\phi=0$, and (b) $\phi=\pi/2$. Insets show the induced waveguides. Propagation of time-averaged intensity with nonlinearity "on" demonstrating statistical nonlinear diffraction (c), and with nonlinearity "off" showing linear diffraction (d). Note that the x scale in (d) is 2.5 times larger than in (a)–(c). (e) Normalized time-averaged intensity profiles at the input face (dotted), and at the output face with (dashed) and without (solid) nonlinearity. (f) Calculated power spectrum of the beam (solid) and probability distribution of the transverse momentum (dashed).

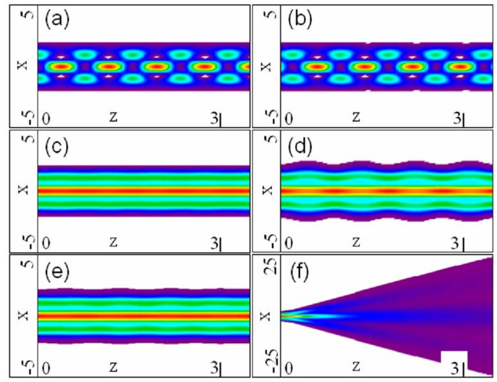

FIG. 2. (Color online). Propagation of random-phase beams consisting of Gauss-Hermite waves 0 and 2. Single frame beam propagation when (a) $\phi=0$, and (b) $\phi=\pi/2$. Propagation of the time-averaged intensity when the incident waves coincide (c) [do not coincide (d)] with the guided modes of the waveguide demonstrating a soliton [breather]. (e) Propagation of the time-averaged intensity of an incoherent beam demonstrating an incoherent soliton. (f) Propagation of the time-averaged intensity with nonlinearity "off" showing linear diffraction. The x scale in (f) is five times larger than in (a)–(e).

with a fast fluctuating directionality. Consequently, the time-(ensemble-) averaged beam [Fig. 1(c)] broadens due to statistical nonlinear diffraction. For comparison, we simulate the linear propagation of the time-averaged intensity [Fig. 1(d)]. After some distance [shown in Fig. 1(e) for $z=3$], the statistical nonlinear diffraction (dashed) leads to a different beam profile compared to the profile of the linearly diffracting beam (solid), although the two intensity maxima in both cases coincide. The spatial power spectrum and the probability distribution of the transverse momentum are plotted in Fig. 1(f). The calculated profiles [Fig. 1(f)] resemble the calculated linear and nonlinear diffraction profiles [Fig. 1(e)]. This shows that the far-field time-averaged intensity structure of the statistical nonlinear diffraction indeed corresponds to the probability distribution of the transverse momentum of the incident beam.

An incoherent soliton forms in our system if the statistical nonlinear diffraction is eliminated, i.e., $p(\bar{k}_x)=\delta(\bar{k}_x)$, which for a wave given by Eq. (4) occurs whenever $n\neq m\pm 1$. Figure 2 shows an example of a soliton comprising of modes 0 and 2. Figures 2(a) and 2(b) show two representative frames: $\varphi=0$ [Fig. 2(a)] and $\varphi=\pi/2$ [Fig. 2(b)]. Now, in every time frame, the beam not only self-traps, but is also always propagating exactly on axis. The ensemble average is shown in Fig. 2(c), demonstrating the stationary propagation of the time-averaged envelope. Figure 2(c) is a representative multimode soliton occurring when the uncorrelated coherent waves of the incident light coincide with the guided modes of the induced waveguide. When the guided modes do not exactly coincide with the uncorrelated coherent waves of the incident light, the waveguide modes are excited with some correlation among them. Consequently, "beating" among the (partially correlated) guided modes will now lead to oscillations of the envelope along propagation. Such an example is shown in Fig. 2(d), where the modal powers of the incident beam are 20% smaller than those of Fig. 2(c). This results in a 20% shallower induced waveguide, supporting guided modes with a different structure than the structure of the incident beam. Next, we study the case by where the modal powers fluctuate randomly. In this case we calculate the propagation of the time-averaged beam via the Monte Carlo method. Figure 2(e) shows the propagation of a beam in which each modal power has a Gaussian distribution with an average $\langle P_n\rangle=\langle P_m\rangle=2\sqrt{\pi}\sigma^3$ [values of Fig. 2(c)] and standard deviation $\sigma_n=0.2\langle P_n\rangle$. As shown there, the beam self-traps into an incoherent "soliton breather". For comparison, Fig. 2(f) shows the linear diffraction of the time-averaged beams.

As a last example, consider a quasithermal light beam propagating in our system. Obviously, quasithermal light excites consecutive modes of the induced waveguide, hence statistical nonlinear diffraction is always present. However, for highly incoherent beams, the number of excited modes is very large and hence the ratio of consecutive pairs to inconsecutive pairs can be very small. In this case the contribution of statistical nonlinear diffraction to the propagation of the time-averaged beam is very small. For example, consider a beam consisting of 50 coherent waves

$$\Psi(x,t,z=0)=\frac{\exp(-x^2)}{\sqrt[4]{2\pi}}\sum_{n=0}^{n=49}\sqrt{\frac{P_n(t)}{2^n n!}}H_n(\sqrt{2}x)\exp[i\varphi_n(t)]$$

incident upon the nonlinear medium. φ_n are statistically independent random variables. The modal powers $P_n(t)$ are random variables having a Gaussian distribution with $\langle P_n\rangle = P\exp(-n/\Delta)/\sum_{n=0}^{n=49}\exp(-n/\Delta)$, $\Delta=25$, $P=4\sqrt{\pi}\sigma^3$, and $\sigma_n=0.2\langle P_n\rangle$. Figures 3(a) and 3(b) show the propagation of two realizations of the incident beam. In each frame the beam is self-trapped, yet the beam in different frames is propagating in different directions, producing the statistical nonlinear diffraction of the time-averaged envelope [Fig. 3(c)]. However,

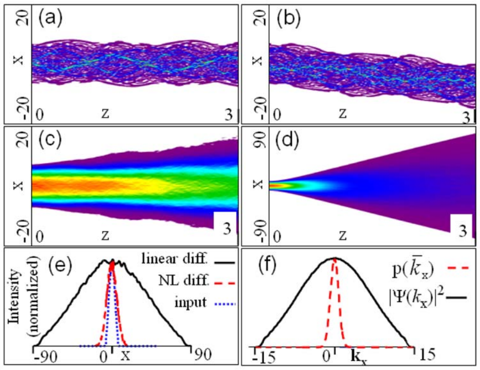

FIG. 3. (Color online). Propagation of an incoherent beam from a quasithermal source. (a) and (b) propagation of individual frames. Propagation of the time-averaged intensity with nonlinearity on demonstrating self-trapping with a small statistical nonlinear diffraction (c), and with nonlinearity off showing linear diffraction (d). The x scale in (d) is 4.5 times larger than in (a)–(c). (e) Normalized profiles of the time-averaged intensity at the input face (dotted), and output face with (dashed) and without (solid) the nonlinearity. (f) Time-averaged power spectrum of the beam (solid) probability distribution of the transverse momentum (dashed).

since the beam has 50 modes, the statistical nonlinear diffraction is very small compared to the linear diffraction [Fig. 3(d); note the 4.5 times difference in the transverse scales between Figs. 3(a)–3(c) and Fig. 3(d)]. Figure 3(e) shows the sharp contrast between the linear (solid) and the nonlinear (dashed) diffraction effects for the time-averaged normalized intensity. Clearly, the nonlinear broadening is much smaller than the linear broadening. Recalling that for the bimodal beam the statistical nonlinear diffraction is comparable to the linear diffraction [Fig. 1(e)] implies that the ratio between the linear and nonlinear broadening increases as the number of modes increase. That is, for the same value of nonlinearity, the more incoherent the beam, the more stationary its time-averaged intensity. There is no "penalty" for making the self-trapped beam more incoherent in a highly nonlocal nonlinear medium; in fact, lower spatial coherence results in "better" self-trapping (as long as the self-consistency is satisfied). Finally, the power spectrum and the probability distribution of the transverse momentum highlight the similarity between the spectral profiles [Fig. 3(f)] and the calculated diffraction profiles [Fig. 3(e)].

In conclusion, we have studied the propagation of spatially incoherent beams in a fast-responding (instantaneous) nonlocal nonlinear medium. A soliton forms in this system when (i) the beam is self-trapped within a time frame much shorter than the characteristic fluctuation time, and (ii) the transverse momentum of the incident beam is constant in time. When the transverse momentum randomly fluctuates in time, the beam exhibits a new kind of diffraction broadening denoted as statistical nonlinear diffraction.

The authors acknowledge Gershon Kurizki from the Department of Chemical Physics of the Weizmann Institute of Science for useful discussions on nonlocal nonlinearities and for the Israel Science Foundation and the German-Israeli DIP project for financial support.

[1] M. Mitchell *et al.*, Phys. Rev. Lett. **77**, 490 (1996).
[2] M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997); Z. Chen *et al.*, Science **280**, 889 (1998).
[3] For a recent review see M. Segev and D. N. Christodoulides, Opt. Photonics News **13**, 70 (2002).
[4] M. Mitchell *et al.*, Phys. Rev. Lett. **79**, 4990 (1997).
[5] D. N. Christodoulides *et al.*, Phys. Rev. Lett. **78**, 646 (1997); V. V. Shkunov and D. Z. Anderson, *ibid.* **81**, 2683 (1998); A. W. Snyder and D. J. Mitchell, *ibid.* **80**, 1422 (1998); H. Buljan *et al.*, Opt. Lett. **28** 1239 (2003).
[6] Another type of incoherent soliton was discovered by A. Picozzi and M. Haelterman, Phys. Rev. Lett. **86**, 2010 (2001); A. Picozzi, M. Haelterman, S. Pitois, and G. Millot, *ibid.* **92**, 143906 (2004). These solitons exist in local instantaneous nonlinearities, forming by virtue of parametric energy exchanges between several waves. The "parameteric incoherent solitons" are fundamentally different from those we describe in this Communication.
[7] H. Buljan *et al.*, Phys. Rev. Lett. **92**, 223901 (2004); O. Cohen *et al.*, Nature (London) **433**, 500 (2005).
[8] M. Soljacic *et al.*, Phys. Rev. Lett. **84**, 467 (2000); D. Kip *et al.*, Science **290**, 495 (2000).
[9] A. G. Litvak, JETP Lett. **4**, 230 (1966);
[10] D. Suter and T. Blasberg, Phys. Rev. A **48**, 4583 (1993).
[11] E. A. Ultanir *et al.*, Phys. Rev. Lett. **90**, 253903 (2003); E. A. Ultanir *et al.*, Opt. Lett. **29**, 283 (2004).
[12] C. Conti, *et al.*, Phys. Rev. Lett. **92**, 113902 (2004); M. Peccianti *et al.*, Nature (London) **432**, 733 (2004).
[13] L. A. Rivlin, Quantum Electron. **28**, 99 (1998).
[14] H. L. Pecseli and J. J. Rasmussen, Plasma Phys. **22**, 421 (1980); N. N. Rao and P. K. Shukla, Phys. Scr., T **T82**, 53 (1999).
[15] F. Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463 (1999).
[16] A. W. Snyder and D. J. Mitchell, Science **276**, 1538 (1997).
[17] S. K. Turitsyn, Theor. Math. Phys. **64**, 226 (1985); O. Bang *et al.*, Phys. Rev. E **66**, 046619 (2002).
[18] D. Briedis *et al.*, Opt. Express **13**, 435–443 (2005).
[19] C. Rotschild *et al.*, Phys. Rev. Lett. **95**, 213904 (2005).
[20] N. I. Nikolov *et al.*, Opt. Lett. **29**, 286 (2004).
[21] The short time frame corresponds to monitoring the instantaneous intensity, while the long time frame corresponds to monitoring the intensity with a slow (relative to t_c) camera. Both times are defined by the source (via t_c), and are not related to the nonlinear response time.
[22] The transformation of the dimensionless Eqs. (1) and (2) into dimensional units (X,Z) is given by $Z=zc/\omega\Delta n_0$ and $X=xc/\sqrt{2\omega^2 n_0\Delta n_0}$, where c is the speed of light in vacuum, ω is the angular frequency, n_0 is the linear index of refraction, and $\Delta n_0=n_2I_0$ where n_2 is the Kerr coefficient and I_0 is a constant obtained by $I_0=I(x,z,t)/|\Psi(x,z,t)|^2$, where I is the beam's instantaneous intensity.
[23] A. W. Snyder *et al.*, Opt. Lett. **16**, 21 (1991).
[24] A. Yariv, *Optical Electronics in Modern Communications* (Oxford, New York, 1997).
[25] M. Mitchell, *et al.*, Phys. Rev. Lett. **80**, 4657 (1998).